Theory and Design of Fixed Field Alternating Gradient Accelerators

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BNL Accelerator Forum

Outline

- Motivation: what problem are we trying to solve?
- What is a Fixed Field Alternating Gradient Accelerator (FFAG)?
- History of FFAGs
- Theory and Design of FFAGs
    - Scaling FFAGs
    - Non-scaling FFAGs
- Concluding remarks

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Motivation

- Rapid acceleration is desired for many applications
  - High repetition rate
  - Accelerating unstable things (muons!)
- Some applications would like CW beams
- A linac is expensive, especially for higher energies
  - Reduce cost by making many passes through the expensive RF

Motivation (cont.)

- Synchrotron
  - Design a ring with limited momentum acceptance
  - Increase magnetic field in proportion to momentum
    - Transverse phase space looks identical at each energy
    - Only longitudinal dynamics change due to velocity variation with energy
  - Rapid momentum increase requires rapid variation of magnetic field: difficult!
  - Typically take thousands of turns (even hundreds of thousands)
    - Uses very little RF
    - Not “rapid acceleration” by our standards
  - Can’t inject another beam until the current beam is extracted and magnets have ramped back down
  - Time-of-flight varies with energy: often must adjust RF frequency to keep synchronized

Motivation (cont.)

- Recirculating Linear Accelerator
  - Make several passes through same linac
  - Dipoles guide the beam into a different arc on each turn
    - Need to pay to build an arc for each pass
    - Each arc has different optics, which must be matched into the linac
  - Magnetic fields don't vary
  - Dipoles can't separate beams if their energy is too close
    - Limits number of passes (about 4), amount of RF re-use
    - Worse for larger transverse acceptance
  - Beam can be injected at any time: CW operation
  - Hit RF at correct phase by adjusting arc length

Motivation (cont.)

- Cyclotron
  - Magnetic fields don't vary as you accelerate
  - Weak focusing: requires enormous magnets (high dispersion)
  - Isochronous: RF frequency can be kept fixed
  - Tune varies with energy: limits energy range

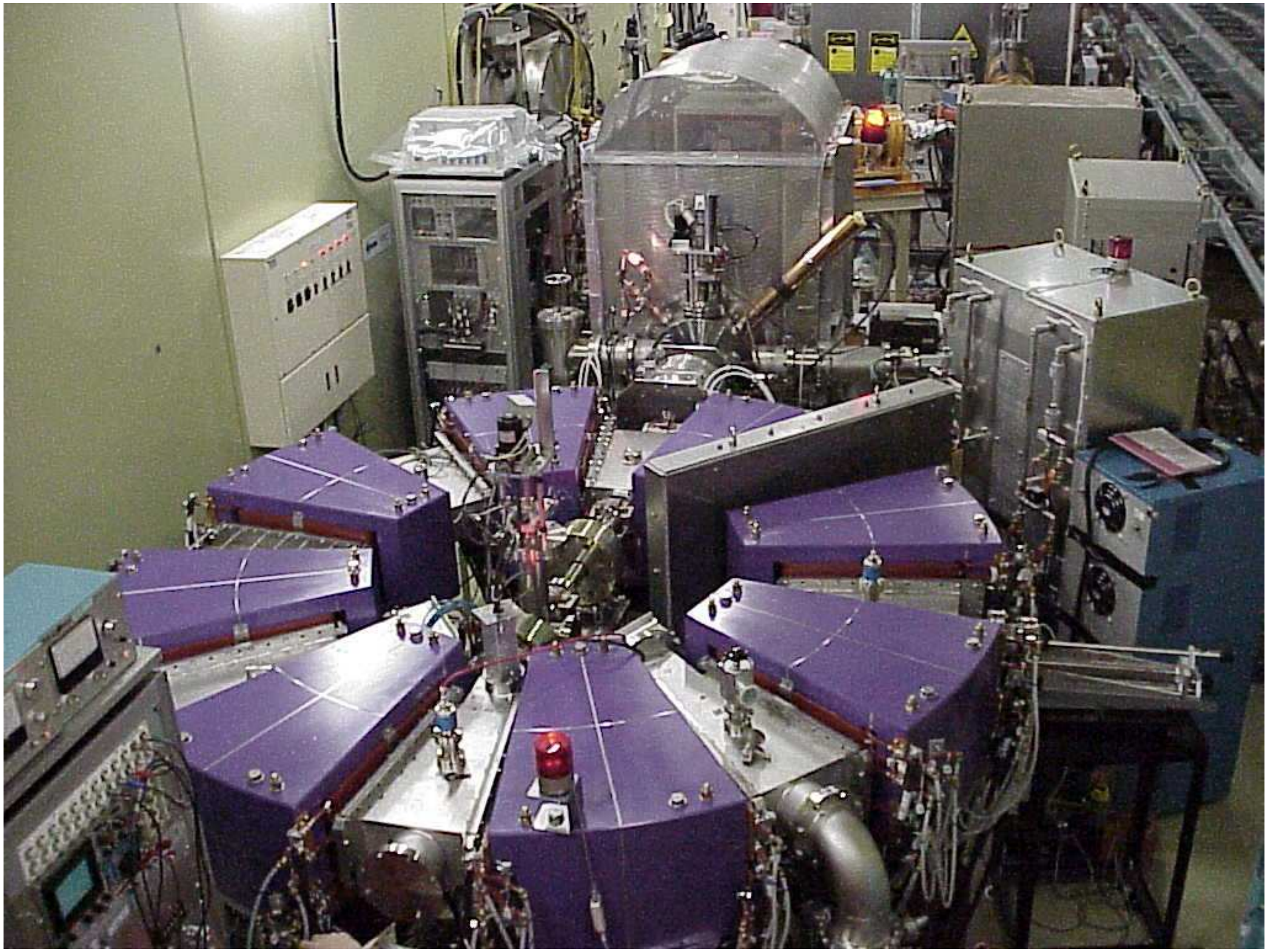
What is an FFAG?

- FFAG stands for **F**ixed **F**ield **A**lternating **G**radient
- Fixed Field
  - Magnetic fields do not vary as you accelerate
  - Therefore, the machine must have a huge energy acceptance
    - Typically at least a factor of two, if not more
- Alternating Gradient
  - Alternate gradients to get strong focusing
  - Smaller magnet aperture than a cyclotron
    - Lower dispersion
    - Smaller beta functions
- Not isochronous: must deal with RF synchronization

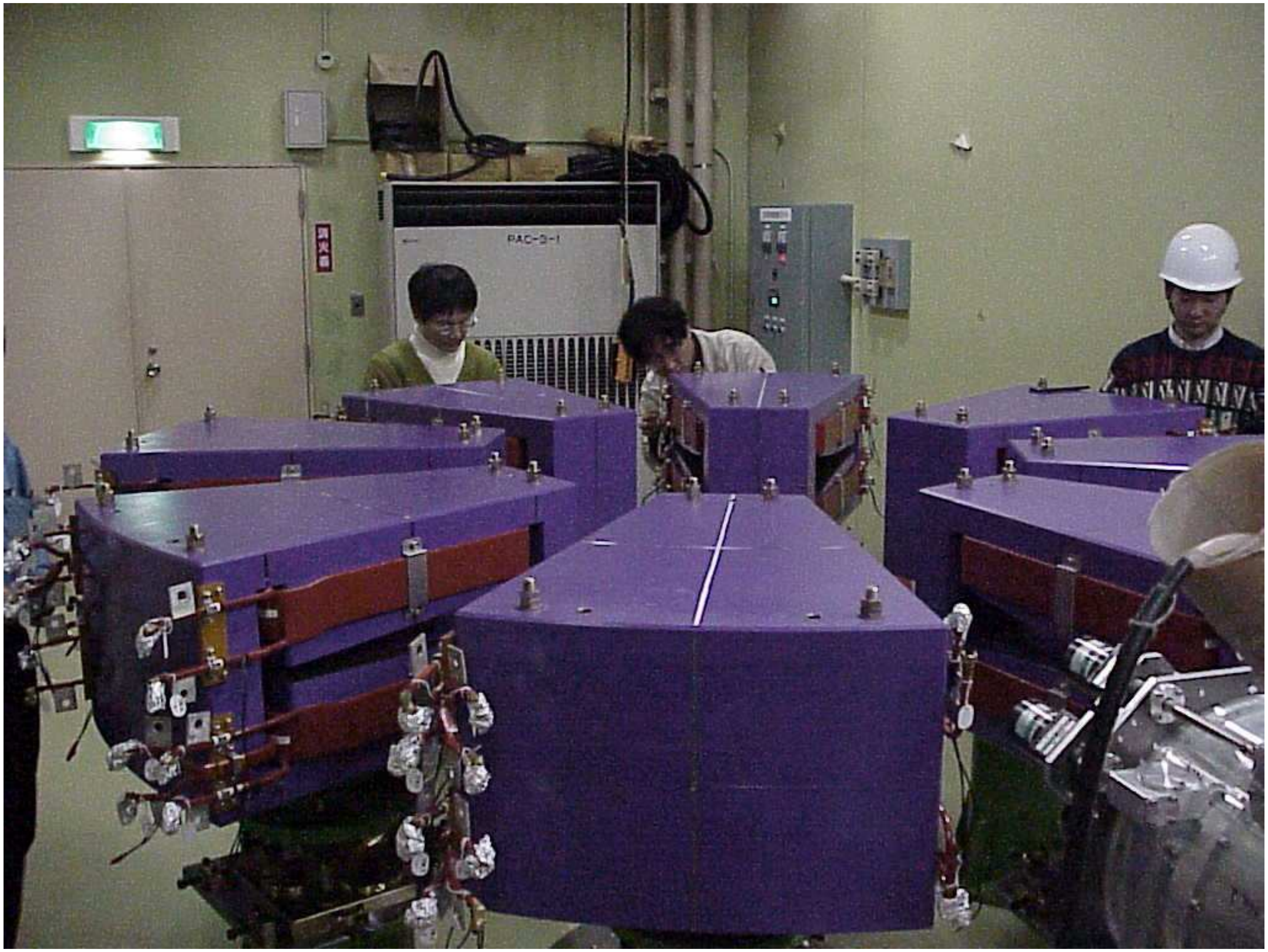
History

- Theory of "scaling" FFAGs: Symon *et al.*, 1956 (Ohkawa 1953?)
- Radial sector electron FFAG built: MURA, 1957
- Spiral sector electron FFAG built: MURA, 1960
- Random mutterings until…
- Johnstone suggests linear non-scaling FODO FFAG (1999)
- Trbojevic suggests nonlinear non-scaling FFAG based on low-emittance lattice design (1999)
- KEK builds "POP" proton FFAG (2000)
- Understanding of longitudinal dynamics develops: Berg, Koscielniak (2001)
- Non-scaling designs converge to triplet design: combination of earlier Johnstone and Trbojevic designs (2002)
- KEK builds 150 MeV proton FFAG (now!)

POP FFAG

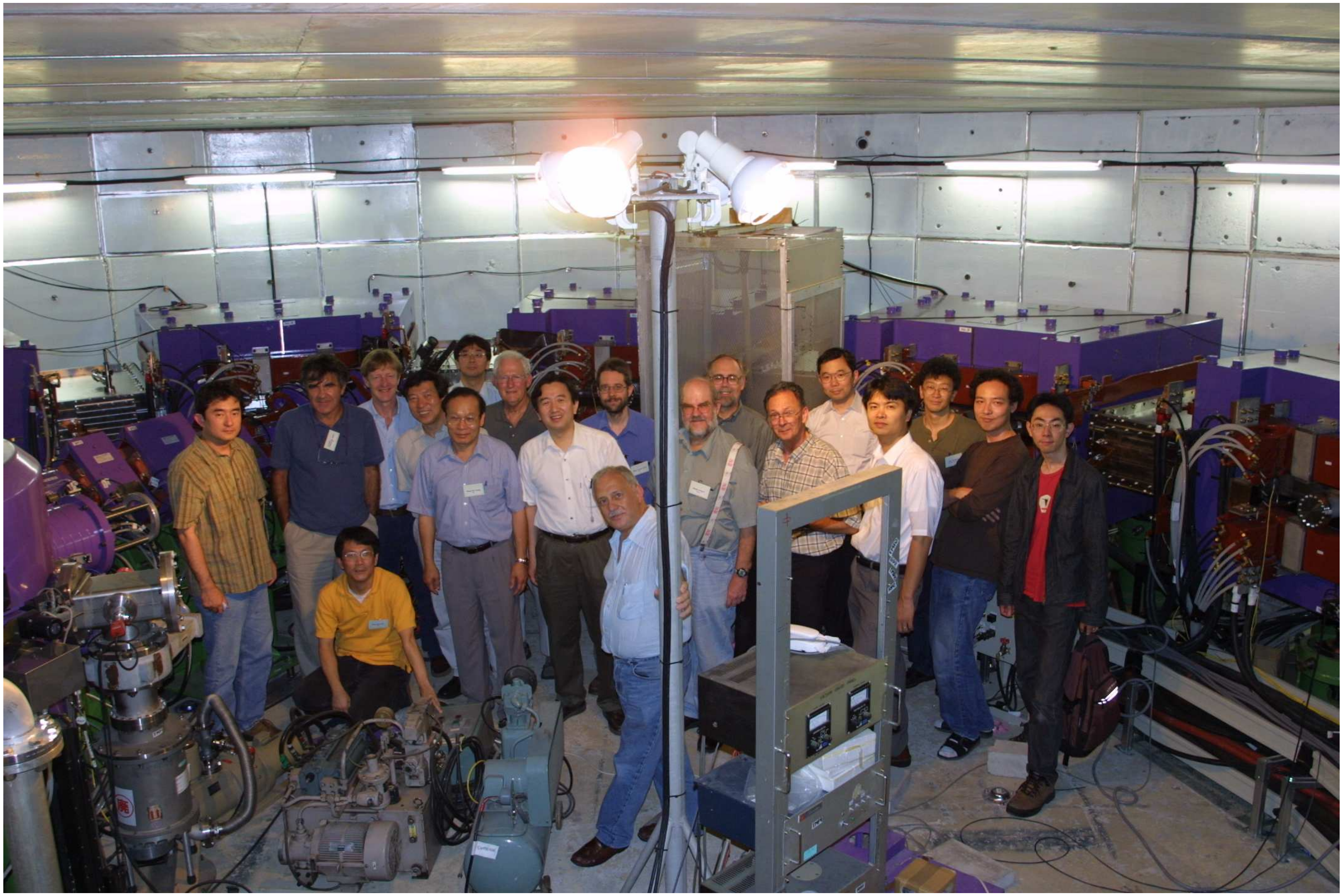


POP FFAG



150 MeV FFAG

150 MeV FFAG



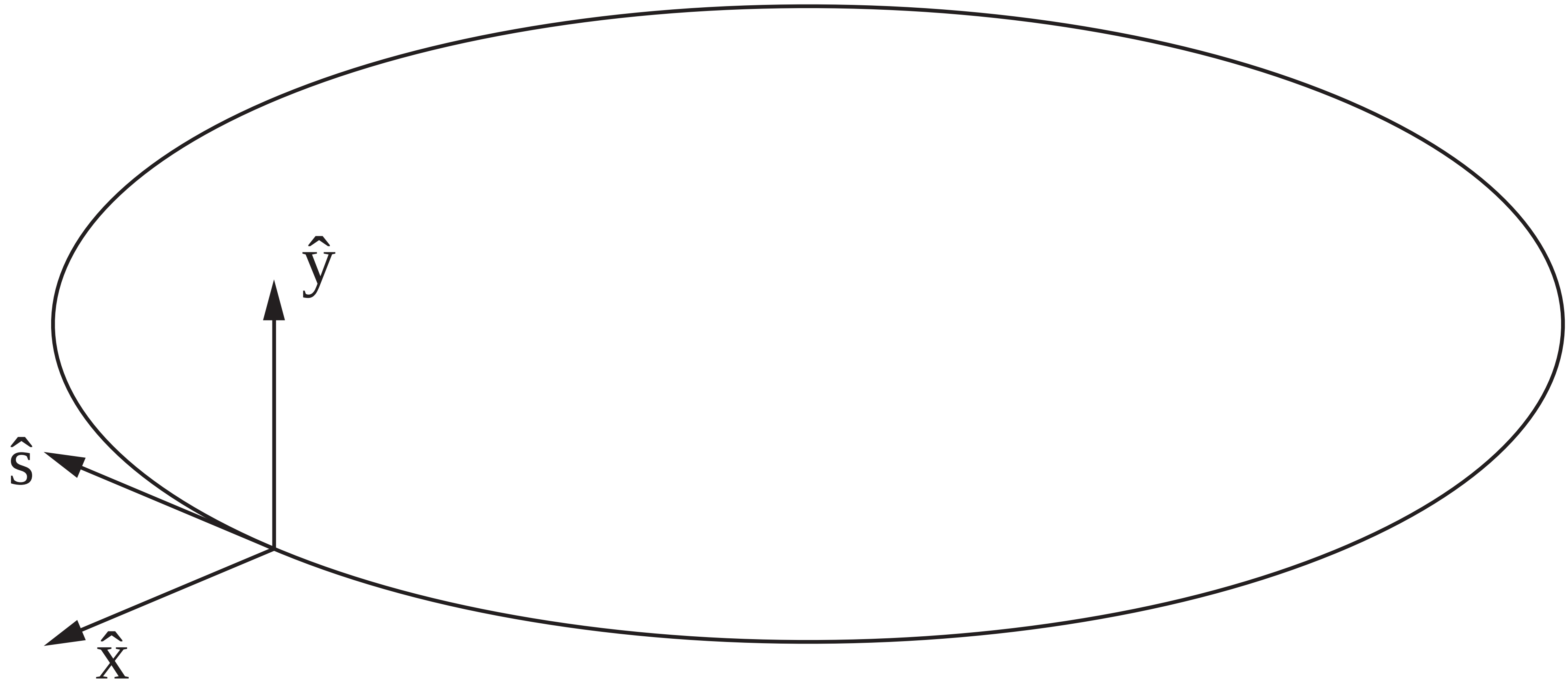
Theory: Scaling FFAGs

- Begin with a circle of radius ρ. This is your reference curve.
  - No particle follows this curve!
  - This curve defines the coordinate system for fields and particles
  - y is the distance perpendicular to the plane of the circle
  - x is the distance from this curve along a radial line in the plane
  - s (the independent variable) is the arc length along the circle
- The magnetic field in the midplane is vertical, and is ($h = 1/\rho$)

$$B_y(x, 0, s) = B_y(0, 0, s)(1 + hx)^k$$

  - Can also have a "spiral angle," which I won't go into here

Coordinate System



Theory: Scaling FFAGs (cont.)

- Maxwell's equations give the magnetic field as

$$
\begin{aligned}
A_x(x,y,s) &= \sum_{n=1} A_{xn}(s)(1+hx)^{k+1-2n}y^{2n}\\
A_y(x,y,s) &= \sum_{n=0} A_{yn}(s)(1+hx)^{k-2n}y^{2n+1}\\
A_s(x,y,s) &= \sum_{n=0} A_{sn}(s)(1+hx)^{k+1-2n}y^{2n}
\end{aligned}
$$

  Note sum of powers of $1+hx$ and y is invariant

- The full accelerator Hamiltonian is

$$
-q(1+hx)A_s - (1+hx)\sqrt{(E/c)^2-(mc)^2-(p_x-qA_x)^2-(p_y-qA_y)^2}
$$

Theory: Scaling FFAGs (cont.)

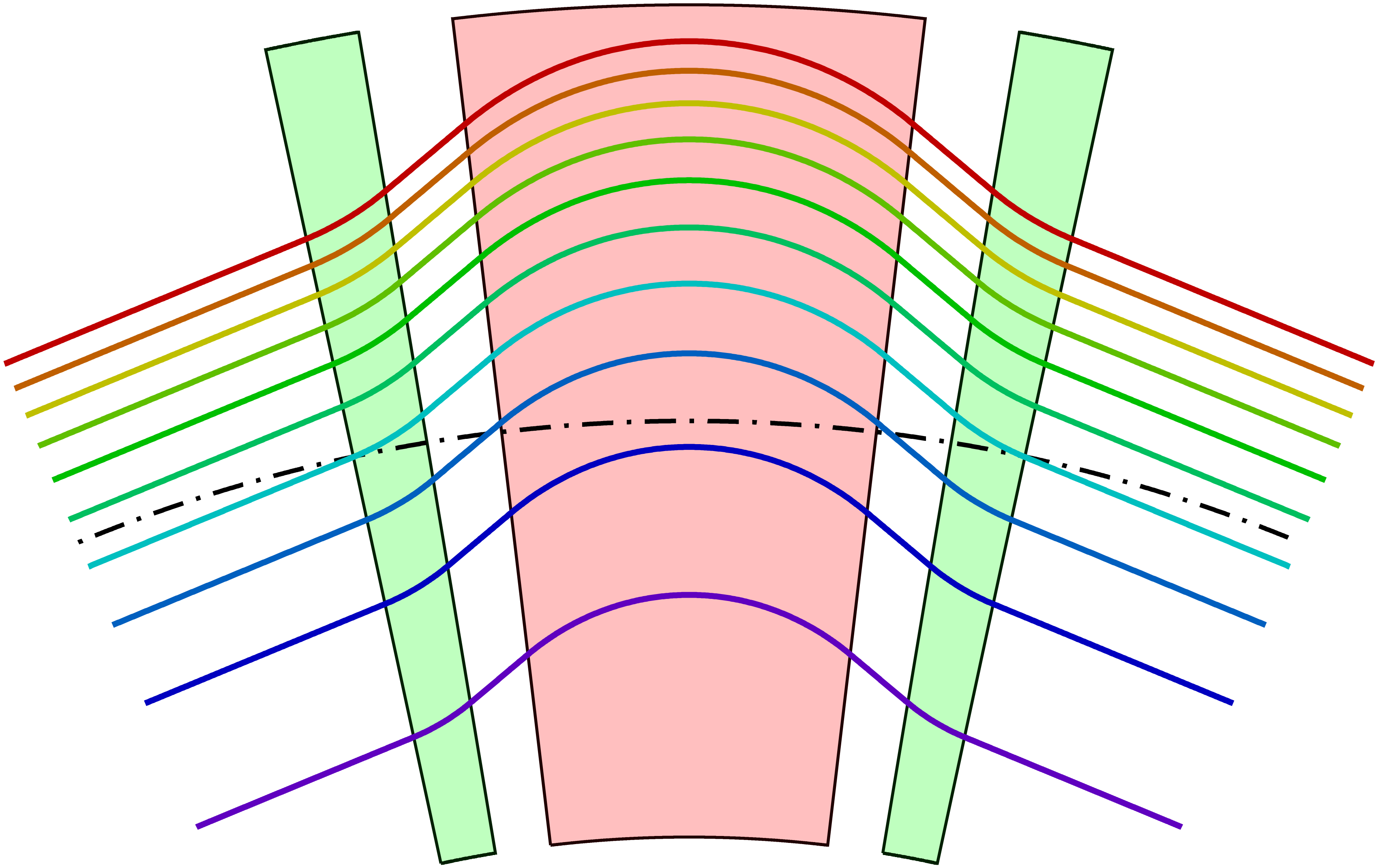
- Peform the transformation $(x, y, t, p_x, p_y) \to (X, Y, T, P_x, P_y)$ given by

$$1 + hX = (1 + hx)\left(\frac{p_0}{p}\right)^{1/(k+1)} \qquad Y = y\left(\frac{p_0}{p}\right)^{1/(k+1)} \qquad T = t\frac{E_0}{E}\left(\frac{p}{p_0}\right)^{k/(k+1)}$$

$$P_x = p_x\frac{p_0}{p} \qquad P_y = p_y\frac{p_0}{p}$$

  - $p^2 = (E/c)^2 - (mc)^2$
  - Result is independent of energy: dynamics at one energy give you dynamics at all energies!
    - Tunes, momentum compaction are constant: $\alpha_C = 1/(k + 1)$
    - Closed orbits geometrically similar
  - Normalized emittance transmitted increases as $(p/p_0)^{(k+2)/(k+1)}$. Slow losses at beginning may be captured.
    - Similar behavior in synchrotron

Scaling FFAG: Closed Orbits



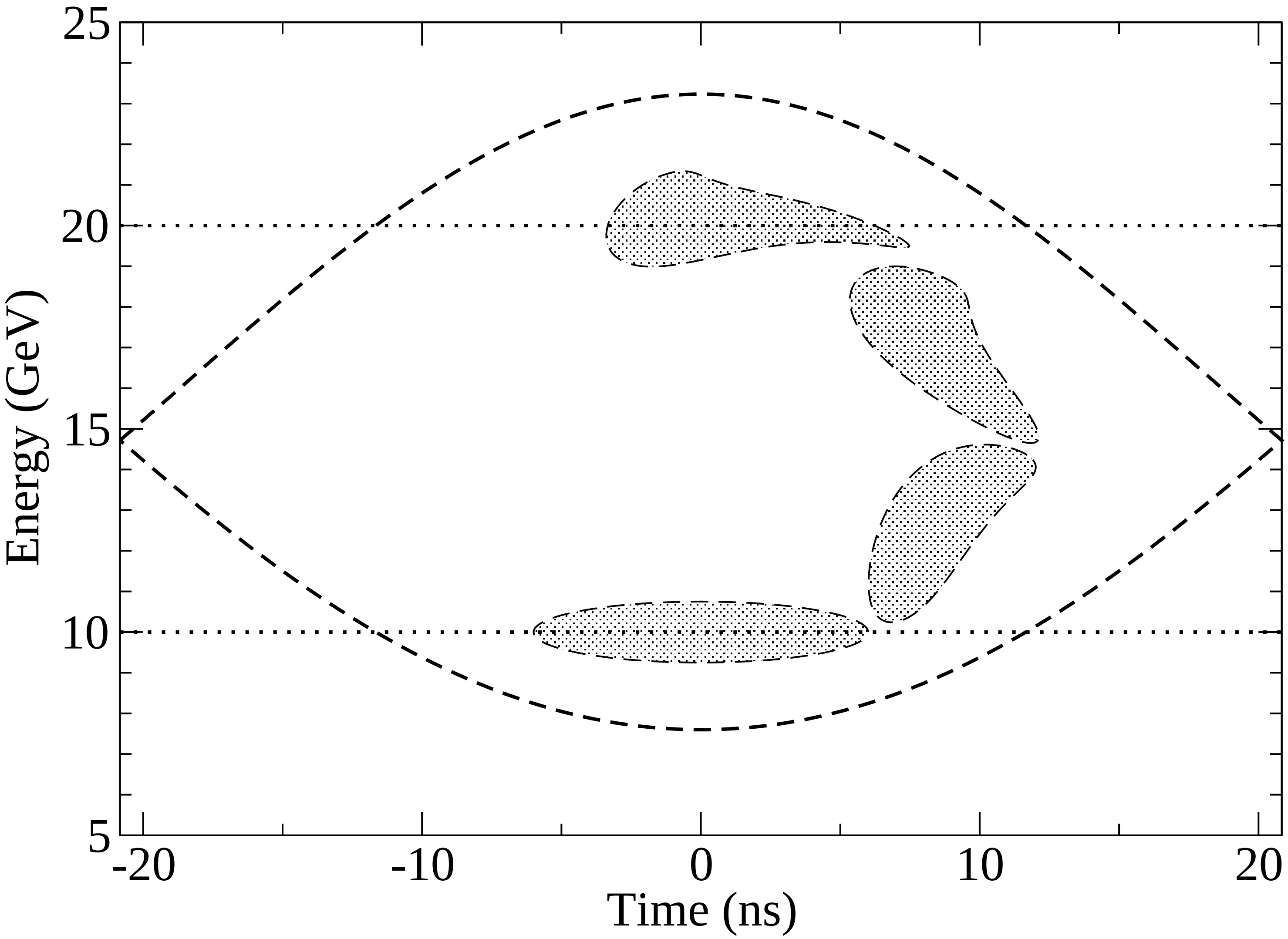
Scaling FFAGs: Longitudinal Dynamics

- Slow acceleration with low-Q cavities: synchronize RF phase with bunch
  - No CW operation: wait for bunch to exit before accelerating next
- Rapid acceleration and/or efficient RF: frequency is fixed
  - Basic problem: time-of-flight varies with energy
  - Solution: undergo half synchrotron oscillation
    - ⋆ RF bucket must cover minimum and maximum energies
    - ⋆ Minimum voltage needed to accelerate

$$V \geqslant \frac{1}{8}\frac{\omega T_0 (\Delta E)^2 [1/(k+1) - 1/\gamma^2]}{\beta^2 E_0} = \frac{\omega \Delta T \Delta E}{8}$$

      V is voltage per cell/ring, T_0 is time to traverse cell/ring, ΔT is range in time to traverse cell/ring. Need extra for nonzero phase space volume.
    - ⋆ Voltage proportional to RF frequency, square of energy range, circumference
      - ➢ More machines with smaller energy ranges may be cheaper
    - ⋆ Relativistic: $k \propto n^2$, so ring voltage $\propto 1/n$

Scaling FFAG: Acceleration



Design Considerations

- For given tunes, aperture proportional to $\Delta E L_{\text{cell}}/n$
  - Shorter cells always better for given tunes
  - More cells give smaller aperture, but more cells: optimize cost
- For given number of cells, k is limited by over-focusing
- Scaling property means you can seek out best working tune
- For fixed-frequency RF system:
  - Non-relativistic: shorter ring requires less voltage.
    - Very low energies or large emittance, betatron size determines aperture: want shortest ring
    - Smaller emittance, tradeoff with voltage and aperture
  - Relativistic: longer ring requires less voltage
    - Tradeoff with number of cells and voltage required: find optimum
    - Aperture variation adds complexity

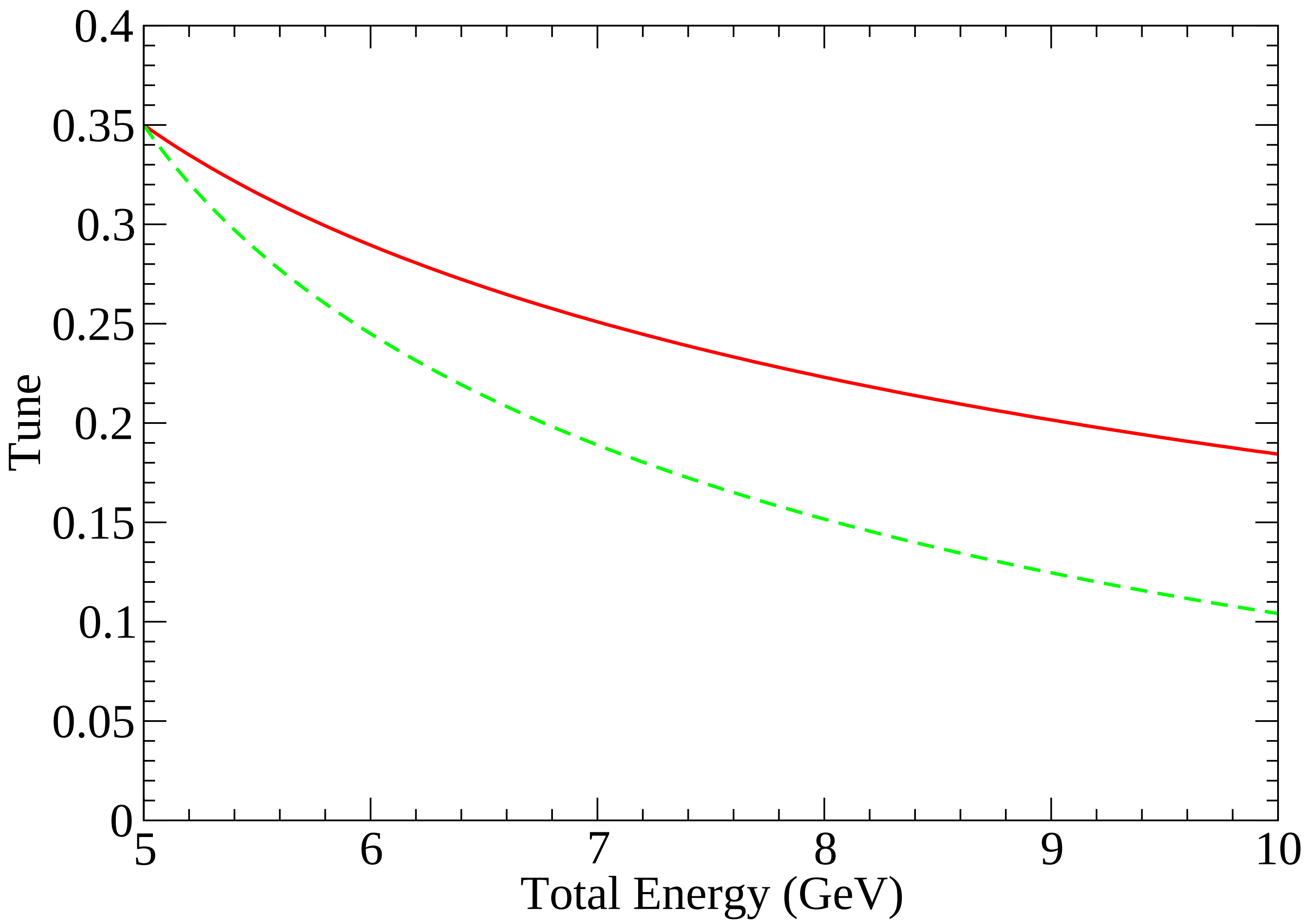
Linear Non-Scaling FFAGs: Motivation

- Problems with scaling FFAGs
  - Require large magnets
  - Highly nonlinear magnets: dynamic aperture
  - Require low frequency and/or large voltages in fixed-frequency case
- Replace nonlinear magnets with linear magnets: dipole-quadrupole combined function
- Make as isochronous as possible to minimize voltage requirement

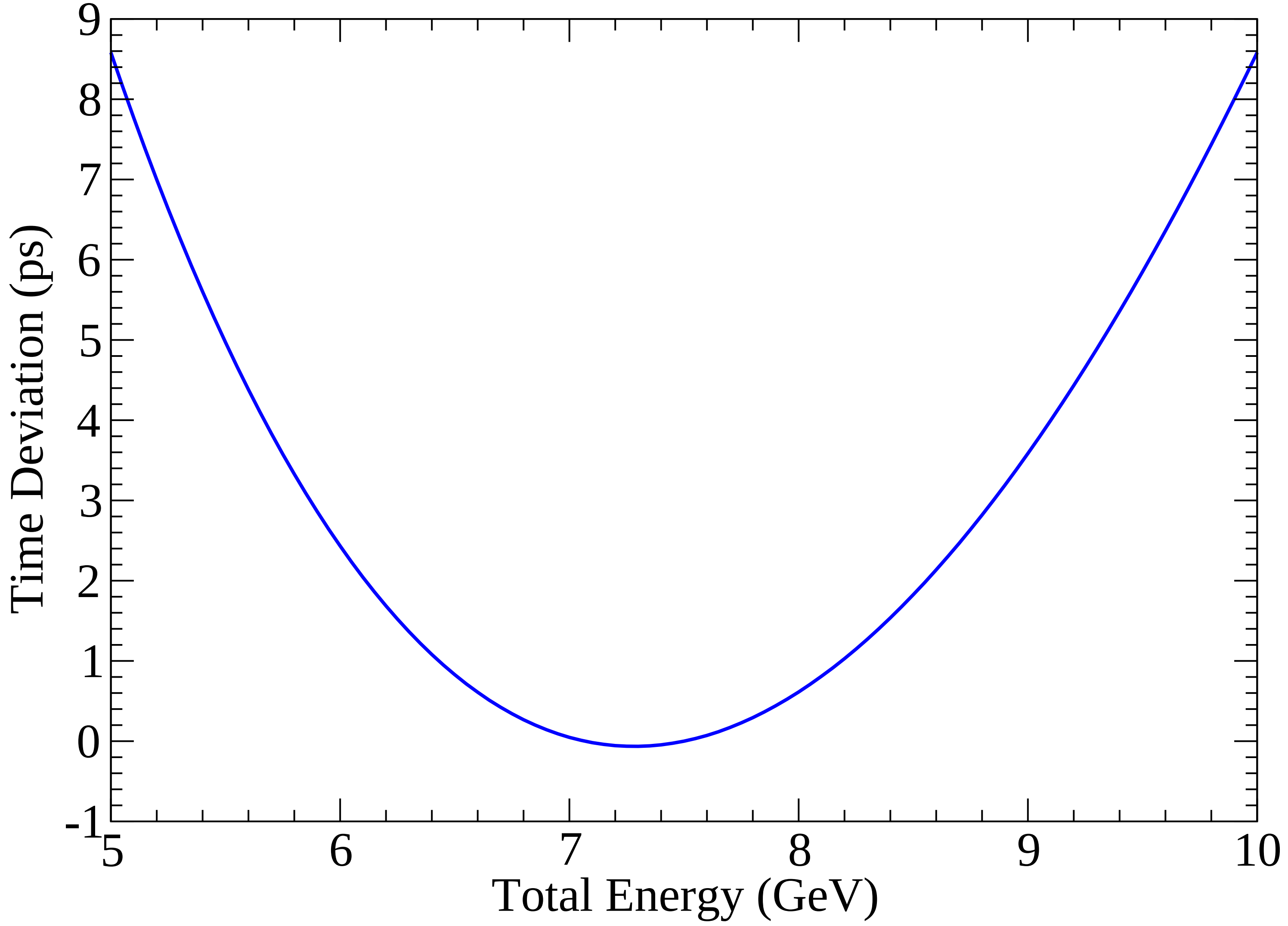
Linear Non-Scaling FFAGs: Theory

- Design with reference orbit following central energy particle.
- Tunes not constant: approach zero at high energy.
  - Pass through many resonances: must accelerate rapidly enough
  - Nonlinear resonances driven weakly by linear magnets
- Must avoid half-integer cell tune at low energy
- High energy, fixed frequency RF: make isochronous near central energy. Time of flight is parabolic vs. energy.
  - Low energy end comes from zigzag; high energy from larger radius
  - Now, $V \geqslant \omega\Delta T\Delta E/24$, ΔT is height of time-of-flight parabola
    - 1/24 compared to 1/8 for scaling
    - ΔT smaller for parabola than for linear for given max slope (half)
  - $\Delta T \propto (\Delta E)^2$, so $V \propto (\Delta E)^3$
    - Even stronger dependence on energy range than scaling

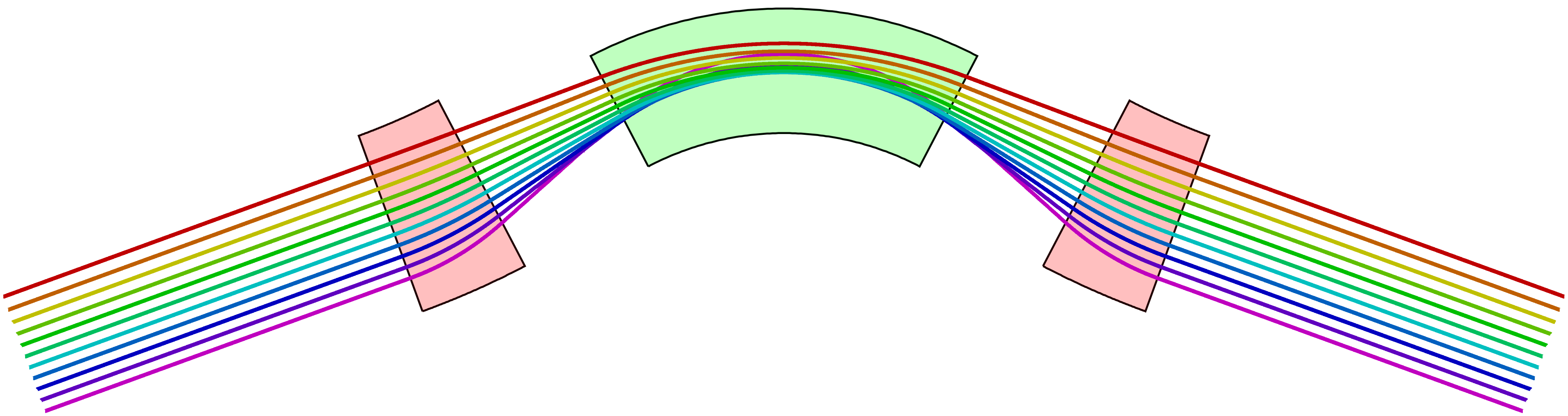
Linear Non-Scaling FFAGs: Tunes



Linear Non-Scaling FFAGs: Time-of-Flight



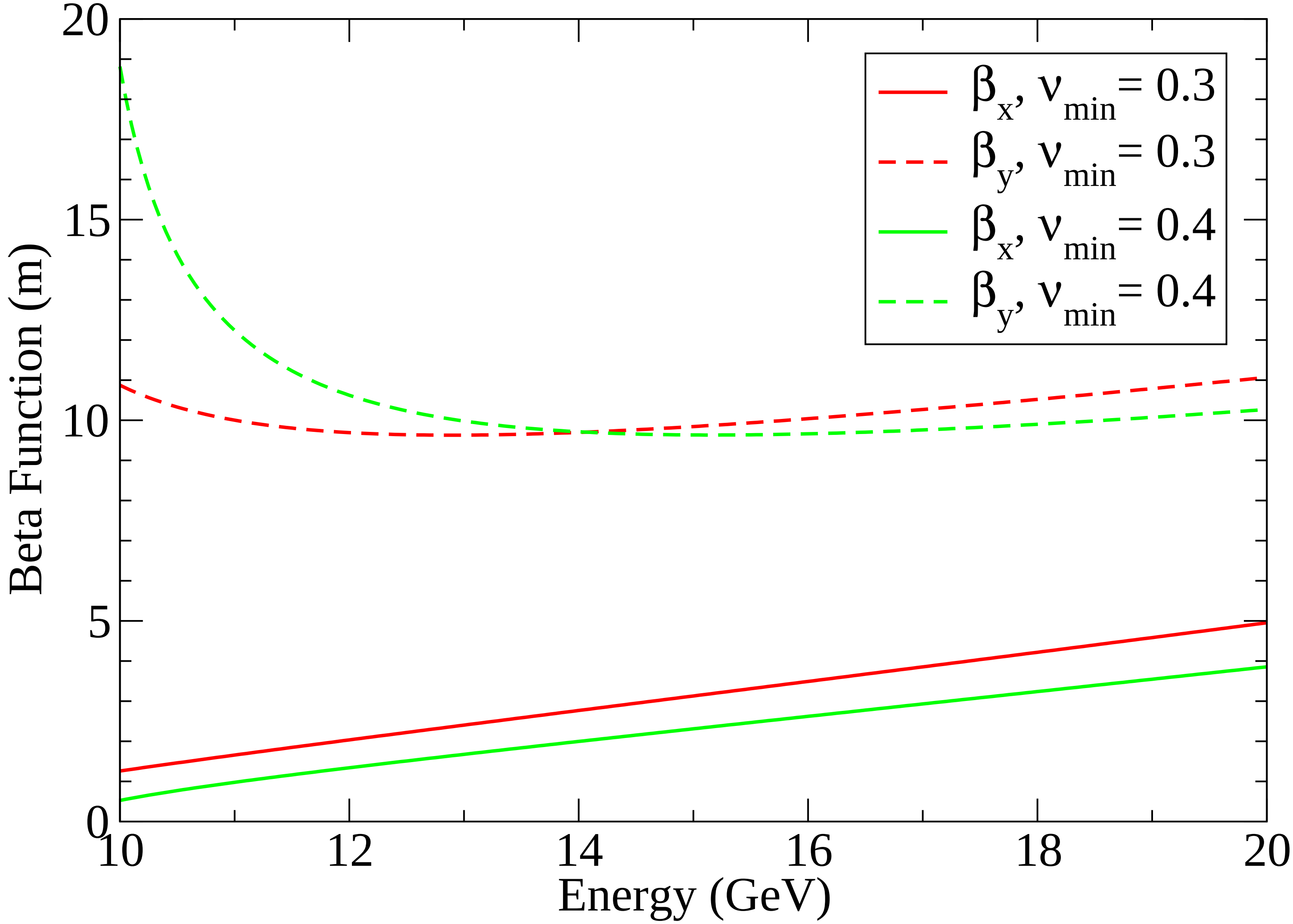
Linear Non-Scaling FFAGs: Orbits



Linear Non-Scaling FFAGs: Theory

- Overlap of orbits reduces magnet aperture
- Other scalings like in non-scaling FFAG
  - $\Delta T \propto \omega L_{\text{cell}}/n$
  - Aperture proportional to $\Delta E L_{\text{cell}}/n$
- Smallest dispersion, ΔT will occur when minimum horizontal betatron function is at bend (Trbojevic)
  - Requires defocusing quads bend forward
  - Leads to FDF triplet configuration
- Raising low-energy tune reduces aperture, ΔT
  - Greater overlap of orbits
  - Cost: sharp rise in betatron function at low energy

Linear Non-Scaling FFAGs: Betas



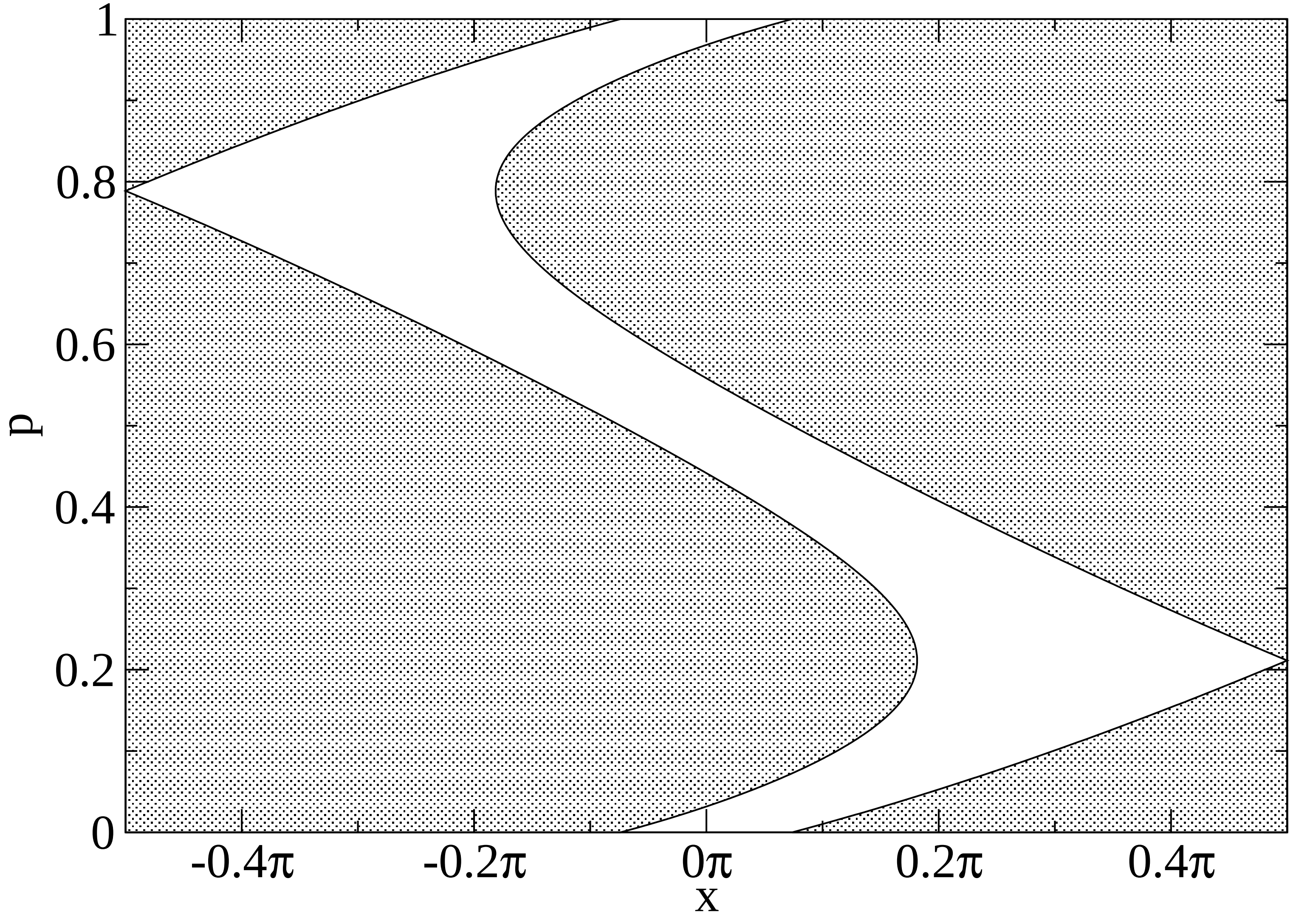
Linear Non-Scaling FFAGs: Resonances

- Large tune variation: cross many nonlinear and imperfection resonances
- Important to maintain symmetry: imperfection resonance
  - Symmetry weakly broken by acceleration
  - Injection section
- Nonlinear resonances: rate of crossing
  - Accelerate quickly enough, cross quickly
  - Highly linear magnets: nonlinear resonances not driven strongly
  - Slow acceleration, will sit near resonances for a long time!
  - Fix: reduce chromaticity by making magnets nonlinear
    - Can backfire: nonlinearities may reduce dynamic aperture (cf. lattices based on low-emittance lattice)
    - Better for small-emittance beams (proton drivers)

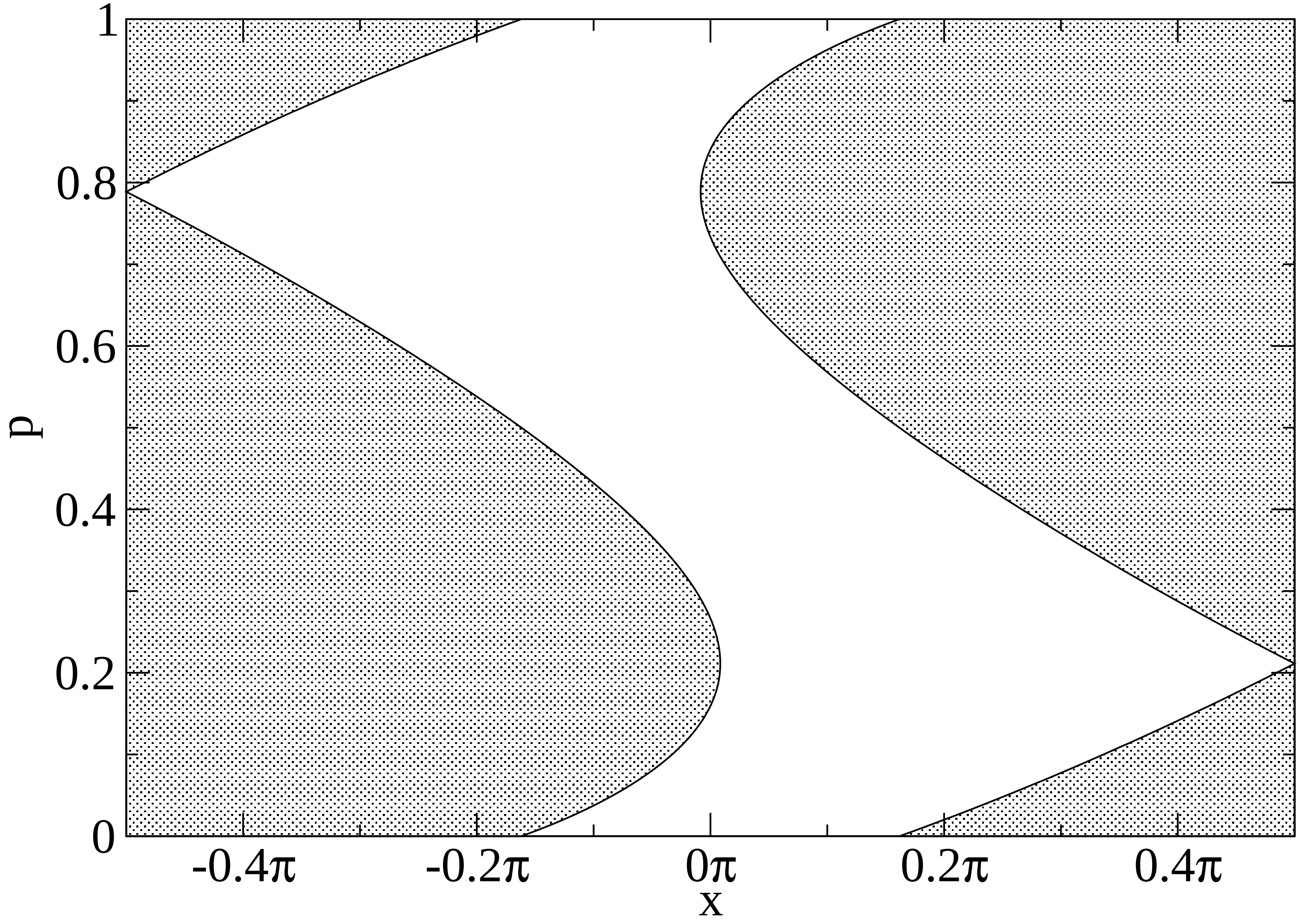
Linear Non-Scaling FFAGs: Longitudinal

- Motion: crossing crest three times
- Motion in channel between stable fixed points
  - Alpha bucket, but outside the bucket
- Width of channel increases with increasing V
- Scaling to dimensionless variables $x = \omega\tau$, $p = (E - E_{\mathrm{min}})/\Delta E$
  - Results depend only on $V/\omega\Delta T\Delta E$ and $T_0/\Delta T$, where T_0 is offset of zero time-of-flight
- Above applies to high-energy systems. Low energy will work more like non-scaling.

Linear Non-Scaling FFAGs: Longitudinal



Linear Non-Scaling FFAGs: Longitudinal



Sample Designs: Muon FFAGs

E_{min} (GeV)	5		10	
E_{max} (GeV)	10		20	
$V/\omega\Delta T\Delta E$	1/8		1/12	
n	90		105	
C (m)	606.918		767.953	
V total (MV)	675.0		787.5	
Cost (PB)	84.5		104.1	
	QD	QF	QD	QF
L (m)	1.612338	1.065600	1.762347	1.275747
r (cm)	14.0916	15.2628	10.3756	12.6256
B_{pole} (T)	2.94697	1.60491	4.30907	2.18390

Sample Designs: Muon FFAGs

- Non-scaling design, “cost optimized”
- Note size doesn’t decrease much with energy range
  - Cell lengths don’t go down in proportion to energy
  - Larger geometric acceptance at lower energies
  - Longitudinal phase space acceptance requirement makes lower energies tougher: energy spread fixed, energy range not
- Even lower energies impractical
  - Scaling or nonlinear machines may work better?

New Ideas

- Increasing degrees of freedom
  - Simplest: scaling FFAG
  - Linear: allow variation of closed orbit
  - Nonlinear: try to control off-energy behavior more carefully
- Add degree of freedom: ramp *some* magnets (Summers)
  - Use high-field magnets to get average behavior
  - Ramp lower-field magnets from negative to positive
  - Program ramp with energy to achieve desired behavior
    - Isochronism
    - Zero chromaticity
  - Good for higher-energy machines, where have more time
  - Much harder design problem!

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Applications

- Muon acceleration
- Proton drivers, other high-intensity proton sources
- Muon phase rotator (PRISM)

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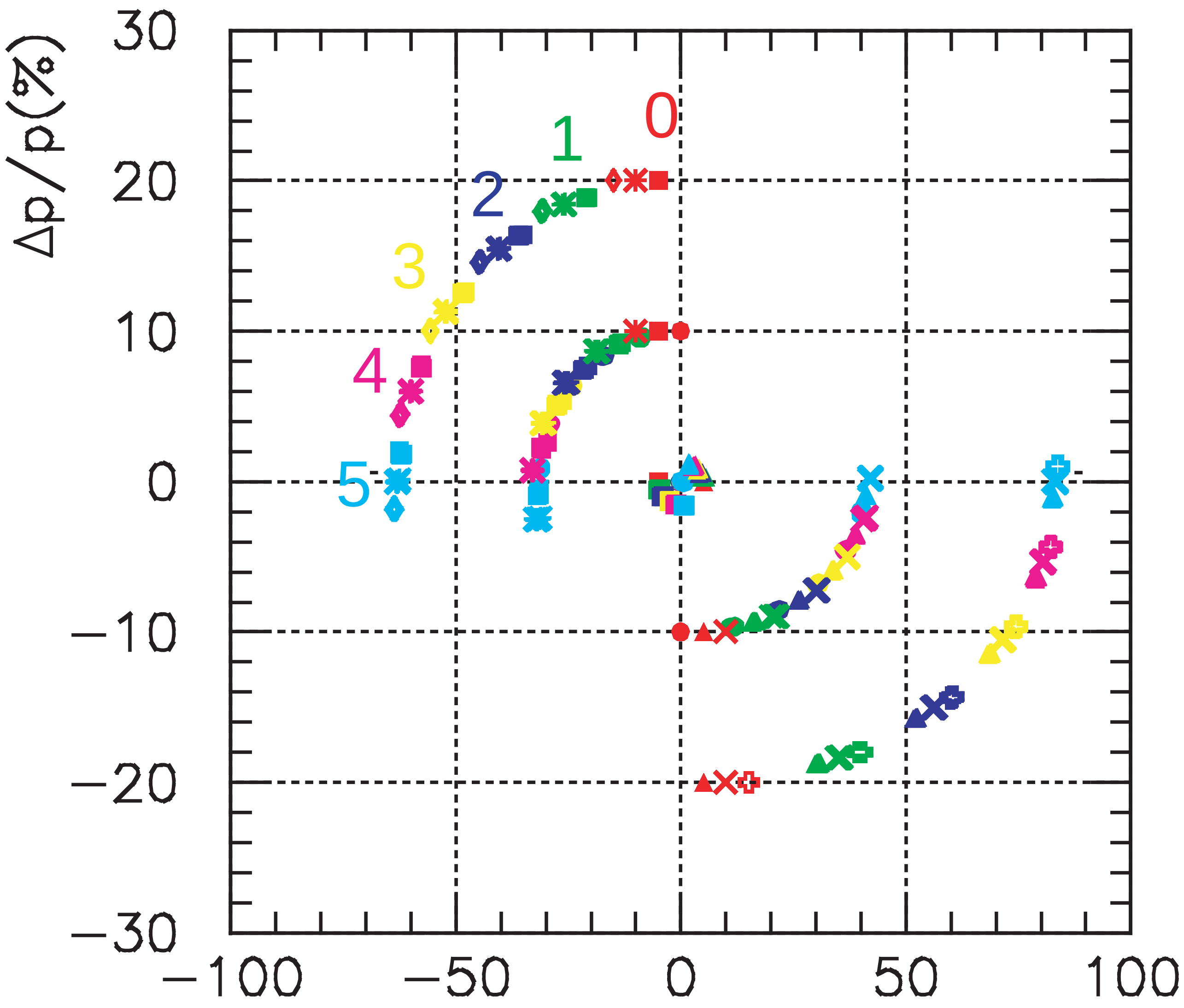
PRISM



Conclusions and Outlook

- There has been a resurgence in interest in FFAGs
- Applications requiring
  - Rapid acceleration
  - CW beams
  - Large energy acceptance
- You now know enough to try designing your own FFAG
- There are still new ideas out there to be explored
  - Nonlinear non-scaling lattices
  - Mixed fixed and ramping magnetic fields
- The big challenge: injection/extraction